

Trade in Commodities and Emerging Market Business Cycles¹

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Abstract

This paper studies the role of the sectoral composition of production and trade in accounting for emerging market business cycles. We document that in emerging economies the production of commodities is a larger share of total production than in developed ones, and that they run larger sectoral and aggregate trade imbalances. We set up a small open economy model that produces commodities and manufactures and trades them with the rest of the world. We contrast the implied business cycle dynamics of two economies that are respectively calibrated to match the observed differences between developed and emerging countries. In the model, shocks to the relative price of commodities lead to much larger fluctuations in output, net exports and TFP in the emerging economy, accounting for the higher volatility that we observe in the data. A key driver of these effects is that emerging economies consume relatively more manufactures than they produce.

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1 Introduction

Business cycles in emerging countries are more volatile than in developed ones. While recent studies have investigated the sources of these differences, the nature of the underlying mechanism remains elusive.¹ In this paper, we study the role of differences in the sectoral composition of production and trade in accounting for the business cycle dynamics of emerging countries.

We document that in emerging economies the production of commodities is a larger share of total production than in developed ones. In addition, emerging countries run larger aggregate trade deficits, on average, driven by large deficits in manufactured goods that are only partially offset by the export of commodities. In developed economies, the average aggregate and sectoral deficits are negligible. Therefore, we argue that changes in the terms of trade are likely to have a bigger impact on emerging economies, as they lead to larger swings in the value of production relative to consumption when sectoral trade imbalances are larger.

To investigate the potential of this mechanism, we study a small open economy with multiple sectors that produce manufactures and commodities. Firms trade these goods internationally taking prices as given from the rest of the world. Aggregate fluctuations are driven by aggregate shocks to the productivity of both sectors as well as by shocks to the relative price of commodities.

We use this framework to contrast the implied business cycle dynamics of two economies that are respectively calibrated to match salient features of developed and emerging countries. We focus on differences along three dimensions. First, we assume that producers of commodities and manufactures differ in their average level of productivity. We calibrate this productivity to match the share of commodities in total output in the two economies. Second, we assume that the share of commodities and manufactures required to produce consumption and investment goods may differ. We calibrate this share to match the sectoral trade deficits and surpluses that we observe in the data. Finally, we assume that the two economies differ in their steady-state level of debt, which we calibrate to match their trade deficits at the aggregate level. To isolate the impact of these structural differences, we keep all other parameters fixed across the two economies.

We find that the structural differences between developed and emerging economies have a significant impact on their business cycle dynamics. While both economies are subject to the

¹Several mechanisms have been proposed as the source of differences in business cycle dynamics. For example, Neumeyer and Perri (2005) focus on the role of international interest rate shocks and financial frictions, while Aguiar and Gopinath (2007) focus on differences in the processes for productivity in emerging and developed economies.

same shock processes, we find that emerging economies feature more volatile business cycles. In particular, real GDP, consumption, and net exports are significantly more volatile, as we observe in the data. Moreover, we find that measured TFP is also significantly more volatile and persistent in emerging economies, as previously documented by Aguiar and Gopinath (2007).

We investigate the specific features of the model that account for these findings. First, we compute impulse response functions for developed and emerging economies in response to each of the shocks. While both economies respond identically to the aggregate productivity shock, the emerging economy features a significantly larger response to the commodity price shock than the developed one. Therefore, we conclude that the higher volatility featured by emerging economies is driven by their higher response to shocks to the relative price of commodities.

We then study the channels that account for the role of commodity prices on the volatility of business cycles. We show that the key feature of the emerging economy that drives the larger response to commodity price shocks is the sectoral trade imbalance. When the emerging economy is calibrated to match the smaller manufacturing trade imbalance of developed economies, while maintaining the other features of emerging countries, we find that the volatility of the key aggregate variables is significantly reduced.

Our findings are closely related to a growing literature that investigates the sources of emerging market business cycles. For instance, Neumeier and Perri (2005), Aguiar and Gopinath (2007), García-Cicco et al. (2010), Chang and Fernández (2013), Hevia (2014), and Comin et al. (2014), provide complementary explanations for the differences in business cycle dynamics featured by developed and emerging economies. We provide a novel mechanism based on the mismatch between the types of goods that are produced and consumed in emerging economies.

Our paper also contributes to the understanding of the role of terms of trade shocks on business cycle dynamics. While Kehoe and Ruhl (2008) show that terms of trade shocks have no effects on measured TFP in standard one-sector models, we show that there are indeed effects in a version of their framework with multiple sectors. Our findings also complement those of Bevan et al. (1993), Kose and Riezman (2001), Baxter and Kouparitsas (2006), and Mendoza, 1995, among others, who study the effect of terms of trade shocks in developing and poor economies.

Finally, our paper is related to a large literature that studies the relationship between differences in the type of goods produced across countries and their economic performance. Previous studies have focused on commodity producers, as in Chen and Rogoff (2003), Raddatz

(2007), Bond and Malik (2009), Furth (2012), Fuentes and González (2012), and Cavalcanti et al. (2014). Van der Ploeg (2011) and Frankel (2012) review a literature that tries to explain why countries with oil, mineral or other natural resource wealth, on average, have failed to show better economic performance than those without. Our paper complements these findings by focusing on the implications for business cycle dynamics.

The rest of the paper is structured as follows. In section 2, we document salient features of developed and emerging economies. In section 3, we set up our model. In section 4, we calibrate the model, present our results, and study the mechanism behind them. In section 5, we present the main conclusions of the paper.

2 Empirical evidence

In this section, we introduce the data that we study throughout the paper and use it to document salient features of developed and emerging economies. We first show that business cycles in emerging economies differ from those in developed ones along a number of dimensions, as previously documented in the literature. We then show that certain cross-sectional features of these two groups of countries also differ markedly: in emerging economies a much larger share of output consists of commodities, and these countries run large trade imbalances (on average) at the sectoral and aggregate levels. In subsequent sections we use a structural model to investigate the link between the cross-sectional features that we document and the business-cycle dynamics of these economies.

2.1 Data

The data that we study is part of the World Development Indicators collected by the World Bank². We restrict attention to annual data from 1970 to 2010. We classify countries into “Emerging” and “Developed” following Uribe and Schmitt-Grohe (2015): countries with average, PPP-converted, GDP-per-capita lower than \$25,000 in 2005 U.S. dollars are referred to as “Emerging”, while the rest are denominated as “Developed”; averages are taken over the period from 1990 to 2009.

We restrict the set of countries to ensure the availability of data across the different dimensions that we study. First, we restrict attention to countries with at least 30 years of consecutive annual observations for each of the business cycle variables that we examine in

²The data is publicly available at <http://databank.worldbank.org/>.

Table 1: Business Cycle Fluctuations: Emerging vs. Developed Economies

	Std. dev. (%)		Std. dev. relative to Real GDP	
	Real GDP	Net Exports / GDP	Consumption	Investment
<i>A. Volatility</i>				
Developed Economies	1.39	1.04	1.04	4.28
Emerging Economies	2.52	2.94	1.74	5.07
<i>B. Corr. with Real GDP</i>				
Developed Economies	1.00	-0.25	0.64	0.83
Emerging Economies	1.00	-0.17	0.50	0.52
<i>C. Autocorrelation</i>				
Developed Economies	0.25	0.07	0.15	0.17
Emerging Economies	0.17	0.00	0.01	0.06

Note: For net exports over GDP we compute the standard deviation in levels. For other variables X we compute the standard deviation of $\log(X)$. All variables are detrended before computing statistics, as described in the text.

section 2.2. In addition, we exclude from the sample any country with less than 50% of observations available in any of the cross-sectional variables that we study in section 2.3. After applying these filters, our final sample consists of 56 emerging economies and 13 developed ones.

2.2 Business Cycle Fluctuations: Emerging vs. Developed Economies

We begin by contrasting the business cycle dynamics of emerging economies with those of developed ones. All variables are *(i)* expressed in real terms by deflating nominal variables with the GDP deflator, *(ii)* seasonally adjusted, and *(iii)* expressed in per capita terms after dividing by population. To identify fluctuations at business cycle frequencies using annual data, we follow Ravn and Uhlig (2002) and de-trend the data applying the Hodrick-Prescott filter with smoothing parameter 6.25.³ Table 1 reports salient features of the business cycle fluctuations of real GDP, consumption, investment, and net exports, between emerging and developed economies. In all cases we report averages across the different groups of countries.

Panel A presents measures of the volatility of these variables. We first observe that, as previously documented in the literature, economic activity in emerging economies is more

³All of our findings are qualitatively robust to alternative de-trending schemes, such as applying the HP-filter with smoothing parameter 100 or examining deviations of the data around a log-quadratic trend.

volatile than in developed ones: the standard deviation of real GDP is 86% higher in emerging countries, and the standard deviation of net exports to GDP is 183% higher in these countries. We also observe that the rest of the variables are considerably more volatile in emerging economies, even relative to the volatility of real GDP. In these countries, consumption is 74% more volatile than real GDP, while in developed economies consumption is as volatile as real GDP. Also, while investment is five times as volatile as real GDP in emerging economies, it is four times as volatile as real GDP in developed countries.

Panel B presents measures of the cyclicity of these variables, as captured by their correlation with real GDP. Along this dimension, we find that the differences between emerging and developed economies are less stark. In both groups of countries it is the case that consumption and investment are pro-cyclical while net exports are counter-cyclical. However, we find that the association of these variables with real GDP is quantitatively less strong in emerging economies.

Panel C presents the autocorrelation of these variables in both groups of countries. We find that the autocorrelations of real GDP, consumption, investment, and net exports are relatively low in both groups of countries. However, these are quantitatively lower in emerging than developed economies for all variables.

These systematic differences in the business cycle dynamics of emerging and developed economies have been previously documented by a number of papers, as noted in section 1. However, while there is broad consensus on the empirical patterns, there is an ongoing debate about the economic mechanisms that drive it. In the next section, we show that these two groups of countries also differ systematically along other dimensions not previously documented in the literature, which suggests the existence of channels complementary to those previously proposed.

2.3 Production of Commodities and Trade Imbalances: Emerging vs. Developed Economies

We now contrast the types of goods produced by emerging and developed economies and their implications for sectoral and aggregate trade imbalances. We abstract from services and partition the goods produced by these countries into two groups: commodities and manufactured goods, where the former consists of goods produced by the agricultural, mining, and fuel sectors. The results are reported in table 2, where the values reported for the different groups of countries correspond to within-group averages.

Table 2: Production of Commodities and Trade Imbalances

	Developed Economies	Emerging Economies
Share of Commodities in Total Value Added	0.42 (0.36, 0.45)	0.67 (0.57, 0.78)
Net Exports of Manufactures / GDP	-0.01 (-0.01, 0.02)	-0.13 (-0.14, -0.004)
Net Exports of Commodities / GDP	-0.004 (-0.03, 0.01)	0.04 (-0.03, 0.11)
Aggregate Net Exports / GDP	-0.01 (-0.03, 0.03)	-0.09 (-0.19, -0.08)

Note: Averages computed for 56 emerging economies and 13 developed countries for the period 1970 to 2010, as described in the text. The values corresponding to the 25th and 75th percentiles, respectively, are in parenthesis.

In the first row of table 2 we report the share of commodities in total value added, for emerging and developed economies. As it can be readily observed, the former exhibit a much larger share of value added accounted by commodities than in the latter. In particular, two thirds of value added in emerging economies is made up of commodities, while only 40% of value added is made up of commodities in developed countries.

To the extent that international trade openness allows emerging economies to be more similar to developed ones in the sectoral composition of consumption relative to the sectoral composition of production, we expect these patterns to reflect in different sectoral trade deficits. This is indeed the case, as observed in the second row of table 2. While imports and exports of manufactures are roughly identical, relative to GDP, in developed economies, there is a sizable mismatch between them in emerging countries. In particular, while emerging economies exhibit, on average, a manufacturing trade deficit equal to 13% of GDP, the average manufacturing trade deficit is only 1% in developed economies. In contrast, while emerging economies are net exporters of commodities, trade of these goods in developed countries is largely balanced, as documented in the third row of the table.

This sectoral pattern of trade imbalances implies the aggregate pattern, reported in the fourth row of table 2. While emerging economies exhibit an aggregate trade deficit equal to 9% of GDP, this deficit is merely 1% on average in developed economies.

These systematic differences in the sectoral composition of production and trade between emerging and developed economies, implies an additional channel that may account for their different business cycle dynamics. In particular, terms of trade fluctuations may have a very

different impact on the business cycle dynamics of economies which produce very similar goods to those they import, relative to economies in which the sets of goods produced and imported are more different. While terms of trade shocks may have a minor impact on the former countries, their impact may be considerably larger in the latter. In the following sections, we investigate the quantitative potential of this mechanism.

3 Model

We study a small open economy model with multiple sectors, where countries produce manufactures and primary goods,⁴ and trade them internationally with the rest of the world. The economy is populated by a representative household, a representative final good producer, and representative producers of primary goods and manufactures.

Time is discrete. Each period there is a realization of a random event s_t , and $s^t = (s_0, s_1, \dots, s_t)$ denotes the history of events up to and including time t . The probability at time 0 of a particular history of events is $\pi_t(s^t)$, and s_0 is given. In general, allocations in period t are functions of the history s^t and of initial values for the capital stock K_0 and asset holdings B_0 , but for notational convenience we suppress their dependence on these initial values.

3.1 Households

We consider a small country with a representative infinitely lived household that derives utility from consumption of final goods $C_t(s^t)$ and leisure $1 - N_t(s^t)$. The utility function is of the constant relative risk aversion (CRRA) class and is given by

$$U_0 = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{[C_t(s^t)^\alpha (1 - N_t(s^t))^{1-\alpha}]^{1-\gamma}}{1 - \gamma} \right] \quad (1)$$

where α is the share of consumption in the consumption-leisure bundle, β is the discount factor, and γ is the coefficient of relative risk aversion. $\mathbb{E}_t[\cdot]$ denotes the expectation operator conditional on information available at time t .

The household supplies labor and capital to firms, trades in an international bond market, and chooses consumption and investment to maximize (1) subject to a budget constraint and

⁴We use the terms “primary goods” and “commodities” interchangeably.

a capital evolution equation, given initial values of the capital stock K_0 and asset holdings B_0 . The budget constraint is given by:

$$p_t(s^t)C_t(s^t) + p_t(s^t)I_t(s^t) + p_t(s^t)q_t(s^t)B_{t+1}(s^t) = w_t(s^t)N_t(s^t) + r_t(s^t)K_t(s^{t-1}) + \Pi_t(s^t) + p_t(s^t)B_t(s^{t-1}) \quad (2)$$

where $I_t(s^t) \geq 0$ is investment, $N_t(s^t) \in [0, 1]$ is the fraction of time spent working, and $K_t(s^{t-1}) \geq 0$ is the capital stock at the beginning of period t . There are no restrictions on the reallocation of resources across sectors, so the wage $w_t(s^t)$ and the rental rate of capital $r_t(s^t)$ are the same in the two sectors. $\Pi_t(s^t)$ denotes the total profits transferred to the household from the ownership of all domestic firms, and $p_t(s^t)$ is the price of the final good.

The household has access to international financial markets where it can trade a non-contingent bond that delivers one unit of the final good next period. $B_{t+1}(s^t)$ is the quantity of such bonds bought by the household in period t , and $q_t(s^t)$ is its internationally given price measured in units of the final good. To ensure the stationarity of bond-holdings, we assume that the bond price is sensitive to the level of outstanding debt as in Schmitt-Grohé and Uribe (2003). Specifically, we assume that it satisfies the equation

$$\frac{1}{q_t(s^t)} = 1 + r^* + \psi \left[\exp \left(- \left(\tilde{B}_{t+1}(s^t) - b \right) \right) - 1 \right] \quad (3)$$

where r^* is the world interest rate, $b \in \mathbb{R}$ is the steady-state level of bond holdings, $\psi > 0$ determines the elasticity of the interest rate to changes in the debt level, and \tilde{B}_{t+1} denotes the aggregate per-capita level of foreign debt. All households are assumed to be identical, so in equilibrium $\tilde{B}_{t+1} = B_{t+1}$. We also impose that $\beta = 1/(1 + r^*)$ to ensure the existence of a steady state.

The household accumulates capital internally by investing final goods subject to a capital adjustment cost. The capital evolution equation is given by

$$K_{t+1}(s^t) = (1 - \delta)K_t(s^{t-1}) + I_t(s^t) - \frac{\phi}{2} \left(\frac{K_{t+1}(s^t)}{K_t(s^{t-1})} - 1 \right)^2 K_t(s^{t-1}) \quad (4)$$

where δ is the depreciation rate of the stock of capital, and changes to the capital stock entail a quadratic adjustment cost governed by $\phi > 0$.

3.2 Firms

There are three types of goods produced in the economy: final goods, manufactures, and primary goods. In each sector there is a representative firm. In this section we describe these firms and the stochastic processes for productivity and prices.

3.2.1 Production of final goods

A representative firm produces final goods using a constant elasticity of substitution (CES) production function. The inputs are manufacturing and primary goods that may be purchased from domestic or international markets. The demands for manufacturing and primary goods in final goods production are denoted by $X_{m,t}(s^t)$ and $X_{p,t}(s^t)$, respectively, and the production function is given by

$$G(X_{m,t}(s^t), X_{p,t}(s^t)) = \left[\eta X_{m,t}(s^t)^{\frac{\sigma-1}{\sigma}} + (1-\eta) X_{p,t}(s^t)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad (5)$$

where parameter σ is the elasticity of substitution between the two inputs,⁵ and η determines the relative weight of manufacturing and primary goods.

The representative final goods producer takes the prices of the two inputs as given and solves the following problem:

$$\max_{X_{m,t}(s^t), X_{p,t}(s^t) \geq 0} p_t(s^t) G(X_{m,t}(s^t), X_{p,t}(s^t)) - q_{m,t}(s^t) X_{m,t}(s^t) - q_{p,t}(s^t) X_{p,t}(s^t) \quad (6)$$

where $q_{i,t}(s^t)$ is the price of input $i \in \{m, p\}$.

The solution to the final good producers' problem determines the price level $p_t(s^t)$, which is given by:

$$p_t(s^t) = \left[\eta^\sigma q_{m,t}(s^t)^{1-\sigma} + (1-\eta)^\sigma q_{p,t}(s^t)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \quad (7)$$

3.2.2 Production of manufacturing and primary goods

In each sector, a representative firm produces using capital and labor with a decreasing returns to scale production function.⁶ For sector $i \in \{m, p\}$ the amount produced $Y_{i,t}(s^t)$, is

⁵For $\sigma = 1$, the final goods production function is Cobb-Douglas.

⁶We assume that firms operate decreasing returns to scale technologies to ensure that, in equilibrium, output is nonzero in both sectors for any combination of sectoral prices.

given by

$$Y_{m,t}(s^t) = Z_t(s^t)K_{m,t}(s^{t-1})^{\theta_k}N_{m,t}(s^t)^{\theta_n} \quad (8)$$

$$Y_{p,t}(s^t) = A_p Z_t(s^t)K_{p,t}(s^{t-1})^{\theta_k}N_{p,t}(s^t)^{\theta_n} \quad (9)$$

where $Z_t(s^t)$ is a time-varying Hicks-neutral level of productivity that affects both sectors, θ_k is the share of capital in production, and θ_n is the share of labor. We assume that these shares are the same across sectors, and that $\theta_k + \theta_n < 1$. In the steady-state, productivity in the manufacturing sector is normalized to 1 while in the primary goods sector it is given by the parameter A_p .

The representative firms take the prices of their output and factor inputs as given and maximize profits by solving

$$\max_{N_{i,t}(s^t), K_{i,t}(s^{t-1}) \geq 0} \pi_{i,t}(s^t) = q_{i,t}(s^t)Y_{i,t}(s^t) - w_t(s^t)N_{i,t}(s^t) - r_t(s^t)K_{i,t}(s^{t-1}). \quad (10)$$

Total profits that are transferred to the households are then given by

$$\Pi_t(s^t) = \pi_{m,t}(s^t) + \pi_{p,t}(s^t). \quad (11)$$

3.2.3 Productivity

The process for the time-varying level of productivity $Z_t(s^t)$ is given by

$$\log Z_t(s^t) = \rho_z \log Z_{t-1}(s^{t-1}) + \varepsilon_{z,t}(s^t) \quad (12)$$

where ρ_z denotes the persistence of productivity and $\varepsilon_{z,t} \sim N(0, \sigma_z^2)$ denotes the shock to productivity.

3.2.4 Prices

We choose the price of manufacturing goods to be the numeraire and set $q_{m,t}(s^t) = 1$ for all time periods t and histories s^t . The small open economy trades manufacturing and primary

goods in international markets and takes the relative price of primary goods $q_{p,t}(s^t)$ as given exogenously. The process for the relative price of primary goods is given by

$$\log q_{p,t}(s^t) = \rho_p \log q_{p,t-1}(s^{t-1}) + \varepsilon_{p,t}(s^t) \quad (13)$$

where ρ_p is the persistence of shocks to the relative price, and $\varepsilon_{p,t} \sim N(0, \sigma_p^2)$ is the innovation.

3.2.5 Market clearing conditions

Market clearing in the manufacturing and primary goods sectors requires that the amount of goods purchased by final goods producers equals the sum of domestic production and net imports of these goods. We let $M_{i,t}(s^t)$ for $i \in \{m, p\}$ be the net amount imported from abroad in sector i . $M_{i,t}(s^t) > 0$ (< 0) implies that goods are imported (exported). The market clearing condition in sector i is then given by

$$X_{i,t}(s^t) = Y_{i,t}(s^t) + M_{i,t}(s^t). \quad (14)$$

The final good is not traded internationally, so the market clearing in the final goods sector simply requires that total production of final goods equals total demand for them by the household for consumption and investment

$$G(X_{m,t}(s^t), X_{p,t}(s^t)) = C_t(s^t) + I_t(s^t). \quad (15)$$

Finally, market clearing in the labor and capital rental markets requires that the amounts of labor and capital supplied by the household equals the total demand for them from the two sectors

$$N_t(s^t) = N_{m,t}(s^t) + N_{p,t}(s^t) \quad (16)$$

$$K_t(s^t) = K_{m,t}(s^t) + K_{p,t}(s^t). \quad (17)$$

3.2.6 GDP

Nominal GDP in the small country is given by the total value added across all sectors, and we let real GDP, $Y_t(s^t)$, be the nominal amount deflated by the price of final goods equation (7)

$$p_t(s^t)Y_t(s^t) = Y_{m,t}(s^t) + q_{p,t}(s^t)Y_{p,t}(s^t) \quad (18)$$

Net exports in the economy $NX_t(s^t)$ is given by

$$NX_t(s^t) = - (M_{m,t}(s^t) + q_{p,t}(s^t)M_{p,t}(s^t)) \quad (19)$$

Using the market clearing conditions in equations (14) and (15), that the representative final goods producer makes zero profits, and the definition of net exports, we can then write GDP following the expenditure approach as

$$p_t(s^t)Y_t(s^t) = p_t(s^t)C_t(s^t) + p_t(s^t)I_t(s^t) + NX_t(s^t) \quad (20)$$

3.3 Definition of equilibrium

Given the law of motion for productivity shocks in equation (12), the international interest rate $r_t^*(s^t)$, and process for the relative price of primary goods $q_{p,t}(s^t)$, an equilibrium in this economy is a set of allocations $C_t(s^t)$, $I_t(s^t)$, $N_t(s^t)$, $N_{i,t}(s^t)$, $K_t(s^t)$, $K_{i,t}(s^t)$, $B_t(s^t)$, $X_{i,t}(s^t)$, $Y_{i,t}(s^t)$, and $M_{i,t}(s^t)$; prices $p_t(s^t)$, $q_t(s^t)$, $w_t(s^t)$, and $r_t(s^t)$ such that: (i) given prices, the households' allocations solve the households' problem; (ii) given prices, the manufacturing and primary goods producers' allocations solve their respective problems; (iii) given prices, the final goods producers' allocations solve the final goods producers' problem; (iv) markets clear.

4 Quantitative implications

In this section, we analyze the implications for business cycle dynamics of differences in the sectoral composition of production and trade. We calibrate two versions of the model presented in section 3 for a developed and an emerging economy. The calibrations capture

the cross-sectional differences in developed and emerging economies that we document in section 2.3. We then compare moments of simulated series from the two economies and present our main findings. Finally, we analyze the forces that drive the business cycles of the two economies by considering impulse response functions and alternative calibrations.

4.1 Calibration

In the calibration of the two economies we aim to highlight the impact of the different sectoral composition of production and trade. We therefore divide the parameter space into two groups. The first group contains all the parameters that we keep fixed for both the developed and the emerging economy. This includes a set of predetermined parameters that are set by picking values from the literature, the parameters governing the process for the relative price of commodities faced by both countries in international markets, and the parameters of the process for aggregate productivity and investment adjustment costs. The second group contains the parameters that are calibrated so the steady states of our two model economies match the cross-sectional features of developed and emerging countries.

4.1.1 Common parameters

Panel A of table 3 shows the set of predetermined parameters. These are common across the two economies, and include the preference parameters, borrowing costs in international financial markets, and most of the technology parameters in the production functions for intermediate and final goods. A period in the model represents a quarter. We set the preference parameters as in Aguiar and Gopinath (2007), so the discount factor β is 0.98, risk aversion γ is set to 2, and the consumption share in the utility function α is 0.36. It follows that the world interest rate r^* that is consistent with a steady state equilibrium is 2%. The parameter ψ that controls the debt elasticity of the interest rate is set to 0.001.

Unlike Aguiar and Gopinath (2007) our model has two sectors. The elasticity of substitution σ between primary and manufactured goods in the production of final goods is set to 1.5 as in Backus et al. (1994). Based on Aguiar and Gopinath (2007) and Midrigan and Xu (2014), we set θ_k to 0.27 and θ_n to 0.58.⁷ Finally, we set the capital depreciation rate δ to 0.05.

⁷In our model with decreasing returns to scale, the production function is equivalent to $y = z(k^\theta n^{1-\theta})^\omega$ with $\omega < 1$. We follow Aguiar and Gopinath (2007) and set $\theta = 0.32$, and we set $\omega = 0.85$ following Midrigan and Xu (2014). These values imply that $\theta_k = \theta\omega = 0.27$ and $\theta_n = (1 - \theta)\omega = 0.58$.

Table 3: Common parameters

A. Predetermined parameters				
Parameter	Value	Source		
β	0.98	Aguiar and Gopinath (2007)		
γ	2	Aguiar and Gopinath (2007)		
α	0.36	Aguiar and Gopinath (2007)		
r^*	0.02	$1/\beta - 1$		
ψ	0.001	Aguiar and Gopinath (2007)		
σ	1.5	Backus et al. (1994)		
θ_n	0.58	See section 4.1.1		
θ_k	0.27	See section 4.1.1		
δ	0.05	Aguiar and Gopinath (2007)		
B. Estimated price process				
Parameter	Value			
ρ_p	0.953			
σ_p	0.060			
C. Calibrated parameters				
Parameter	Value	Target moment	Data	Model
ρ_z	0.553	Autocorrelation real GDP	0.25	0.25
σ_z	0.0082	Standard deviation real GDP	1.39	1.39
ϕ	0.036	Relative std. dev. investment	4.28	4.28

Both the developed and the emerging economy trade primary and manufactured goods in international markets, and the prices they face are taken as given. To capture the fluctuations in the relative price of primary goods we use data from the “Producer Price Index - Commodity Classification” published by the Bureau of Labor Statistics. For commodity prices we follow Gubler and Hertweck (2013) and use the “PPI by Commodity for Crude Materials for Further Processing” index. As they discuss in detail, this index captures much of the variation in commodity prices of alternative indexes, and is available for a longer time period.⁸ For the price of manufactured goods we use the “PPI by Commodity for Finished Goods Less Food & Energy” index. This index is only available starting in 1974, so we estimate the parameters in equation (13) using data from the first quarter of 1974 to the last quarter of 2010. Panel B of table 3 reports our estimates. The estimated process for the relative price of commodities features a high persistence with ρ_p estimated at 0.953, and a

⁸In addition, Gubler and Hertweck (2013) point out that this index is also used by Hanson (2004) and Sims and Zha (2006).

high standard deviation σ_p of 0.060.

Finally, panel C of table 3 reports the values for the persistence ρ_z and the standard deviation σ_z of the productivity process in equation (12) and ϕ which determines the capital adjustment costs in equation (4). These parameters are chosen to match certain features of developed country business cycles. From table 1 we choose to match the standard deviation and autocorrelation of GDP, and the relative standard deviation of investment in developed countries. The table reports these moments for annual data, so we annualize the simulated series from our quarterly model before computing the corresponding moments. As shown in table 3, the model matches the data moments exactly when we set ρ_z to 0.553, σ_z to 0.0082, and ϕ to 0.036. Since our aim is to analyze the impact of the different cross-sectional features of developed and emerging countries, we fix these parameter values also for the emerging economy.

4.1.2 Country-specific parameters

To complete the calibration for the developed economy, we choose three parameters so that the steady-state of that economy matches the cross-sectional features of developed countries reported in table 2.⁹ The steady-state share of commodities in total value added is determined by A_p , the relative productivity of the primary goods sector relative to the manufacturing sector. The share of manufacturing goods in the production of final goods η determines the demand for manufactured goods, and hence the share of manufacturing net exports in GDP. Finally, the level of steady-state bond holdings b determines the ratio of overall net exports to GDP. With A_p set to 0.96, η set to 0.55, and b set to 0.13, the model matches these moments exactly as shown in panel A of table 4. While we don't target the ratio of commodities net exports to GDP, this is implied by the other moments that we target.

Panel B of table 4 shows the corresponding parameters and moments for the emerging economy. When A_p is set to 1.11, η is set to 0.45, and b is set to 1.33, the steady state of the model matches the cross-sectional features of emerging countries. As in the data, the emerging economy has a higher share of commodity production in total value added and runs a larger aggregate trade deficit which is due to a large trade deficit in the manufactured goods sector.

⁹Note that we pick these parameters governing the steady state of the developed economy before we pin down the parameters in panel C of table 3.

Table 4: Country-specific parameters

A. Developed economy				
Parameter	Value	Target moment	Data	Model
A_p	0.96	Commodity share in total value added	0.42	0.42
η	0.55	Manufacturing NX/GDP	-0.01	-0.01
b	0.13	NX / GDP	-0.01	-0.01
B. Emerging economy				
Parameter	Value	Target moment	Data	Model
A_p	1.11	Commodity share in total value added	0.67	0.67
η	0.45	Manufacturing NX/GDP	-0.13	-0.13
b	1.33	Aggregate NX / GDP	-0.09	-0.09

Note: Parameters are calibrated separately for developed and emerging economies to match the moments presented in table 2.

4.2 Results

We solve the model by log-linearizing the equilibrium conditions around the steady state, and solving the resulting system of linear difference equations. The model is then simulated for 164 quarters.¹⁰ We simulate quarterly series, annualize them, and compute the relevant moments for comparison with the moments for annual data presented in table 1. Table 5 reports the averages over 1000 simulations for both the developed and the emerging economy. In addition to the variables considered in table 1, we report moments for employment and TFP in the model.

As panel A of table 5 shows, the business cycles of our calibrated emerging economy are considerably more volatile than those of the developed one. The standard deviations of GDP and net exports as a share of GDP are more than twice as large in the emerging economy. This is the case even though the process for aggregate productivity in the model is the same across the two economies. Therefore, the increased volatility in the emerging country results from the different cross-sectional structure of that economy. It produces and exports more commodities as a share of GDP, while it imports more manufactured goods. In the next section, we show that these differences imply that the emerging economy is more exposed to fluctuations in the relative price of these goods, and this exposure translates into more volatile business cycles.

¹⁰We simulate the model for 1164 quarters starting at the steady state, and drop the initial 1000 quarters before computing any moments.

Table 5: Business Cycle Fluctuations in the Model: Developed vs. Emerging Economies

<i>A. Volatility</i>						
	Std. dev. (%)		Std. dev. relative to GDP			
	GDP	NX/GDP	C	I	N	TFP
Developed	1.39	1.32	0.22	4.28	0.63	0.50
Emerging	3.63	3.21	0.38	3.46	0.58	0.46
<i>B. Correlation with GDP</i>						
	GDP	NX/GDP	C	I	N	TFP
Developed	1.00	0.67	0.60	0.38	0.98	0.99
Emerging	1.00	0.71	0.68	0.37	0.94	0.97
<i>C. Autocorrelation</i>						
	GDP	NX/GDP	C	I	N	TFP
Developed	0.25	0.07	0.57	-0.36	0.29	0.24
Emerging	0.72	0.41	0.86	-0.02	0.72	0.70

Note: For net exports we compute the standard deviation of NX/GDP. For other variables X we compute the standard deviation of $\log(X)$ and divide by the standard deviation of $\log(\text{GDP})$.

As is the case with GDP, we also find that consumption, investment, employment and measured TFP are all more volatile in the emerging economy.¹¹ However, as table 5 shows, the volatility of these variables relative to GDP is similar to the relative volatilities in the developed country. As shown in Panel B, the cyclicity of all of these variables is also fairly similar across the two economies. Relative to the data, our model does not generate the countercyclicality of net exports over GDP in either economy, and it understates the procyclicality of investment.

Finally, the exposure of the emerging economy to the persistent process for the relative price of commodities translates into very persistent business cycles in that economy. In the emerging economy, the autocorrelation of GDP, net exports as a share of GDP, and consumption are all considerably higher than in the data.

In summary, our model isolates the effect of the different cross-sectional features of emerging and developed countries, and shows that these differences alone imply considerably more volatile business cycles in emerging economies.

¹¹TFP is measured following a standard growth accounting approach. In particular, TFP is implicitly defined by $\text{Real GDP} = \text{TFP}K^{\theta_k}N^{\theta_n}$, where $\text{Real GDP} = \text{Nominal GDP}/P$.

4.3 Mechanism

We now investigate the mechanism that underlies the differences in business cycle dynamics that we find between developed and emerging economies. To do so, we first study the response of these economies to one-time productivity and commodity price shocks. We then investigate the cross-sectional features that account for the differences in business cycle dynamics across the two economies.

4.3.1 Impulse response functions

We first examine the impulse response functions of aggregate variables in the developed and emerging economies to (i) an aggregate productivity shock, and to (ii) a shock to the relative price of commodities. Specifically, figures 1 and 2 plot the response of key aggregate variables of the model to one-time one-standard-deviation orthogonal shocks to productivity and the relative price of commodities, respectively. The dynamics of the shocked variables are plotted in the bottom-right panel of each of the figures.

In figure 1, we find that the response to an aggregate productivity shock is consistent with earlier findings in the literature. An aggregate productivity shock leads to an increase in output and consumption. There is also an increase in labor which further increases these responses. Moreover, the shock to productivity leads to a sharp increase in investment that is financed through debt, as reflected by the decrease of net exports on impact. The increase is short-lived due to the low persistence of productivity, and net exports increase to service the debt. Finally, output increases symmetrically across sectors, as is also the case for measured TFP.

Notice also that the developed and emerging economies respond in exactly the same way to aggregate productivity shocks. Therefore, we conclude that any differences in the business cycle dynamics featured by these economies, in the model, are not driven by shocks to aggregate productivity.

In figure 2, we find that shocks to the relative price of commodities have a substantial impact on key aggregate variables of the emerging economy, but their impact on the developed economy is significantly lower. First, note that in both economies an increase in the relative price of commodities leads to a sectoral reallocation of production: production of commodities increases, while production of manufactures decreases. This large reallocation of production, however, is accompanied by very different responses across the two economies. In the emerging economy, there is a significant increase of output, consumption, investment,

and imports (thus the decrease of net exports). There is also an increase in measured TFP, as the amount of final goods that can be produced per unit of capital and labor employed increases when the relative value of the imported good decreases. In contrast, aggregate variables in the developed economy remain largely unchanged.

Our conjecture, which is confirmed in the next subsection, is that shocks to the relative price of commodities have a very different impact on economic aggregates in the presence or absence of sectoral trade imbalances. In the emerging economy, where there are sectoral trade imbalances, a positive shock to the relative price of commodities increases the price of the good that is exported and decreases the price of the good that is imported. These changes have a positive wealth effect that increases economic activity. In contrast, in the developed economy, where sectoral trade is balanced, an increase in the relative price of commodities does not have a wealth effect, as the positive impact of an increase in the value of domestically produced commodities is almost exactly offset by the increase in the price paid to consume commodities.

4.3.2 Role of cross-sectional differences

In this section, we investigate the features of the calibration that drive the differences between the business cycles of developed and emerging economies. To do so, we contrast the business cycle dynamics implied by our calibrated economies with counter-factual ones that eliminate some of the cross-sectional differences between them. To ease the exposition, we restrict attention to sectoral trade imbalances, since we found it to be the key channel driving our results. Specifically, we show that most of the differences in the volatility of business cycle dynamics in the model can be explained by the trade imbalances across sectors in the emerging economy. Thus, this economy is more exposed to shocks to the relative price of commodities in terms of manufactures because the consumption of manufactures is higher than its production.

Table 6 presents the business cycle moments obtained for two alternative calibrations of our model. In each panel, the first row, labeled “Emerging”, corresponds to the benchmark calibration for the emerging economy that we discuss above. In the second row, “Emerging balanced”, we target the share of commodities in total output and aggregate net exports to GDP of an emerging economy, but instead we target the net exports of manufactures to GDP of a developed economy. The third row, “Developed”, corresponds to the benchmark calibration for the developed economy described above. Finally, in the fourth row, “Developed imbalanced”, we still target the share of commodities in total production and aggregate

Figure 1: Impulse Response to a One-Standard-Deviation Shock to Productivity

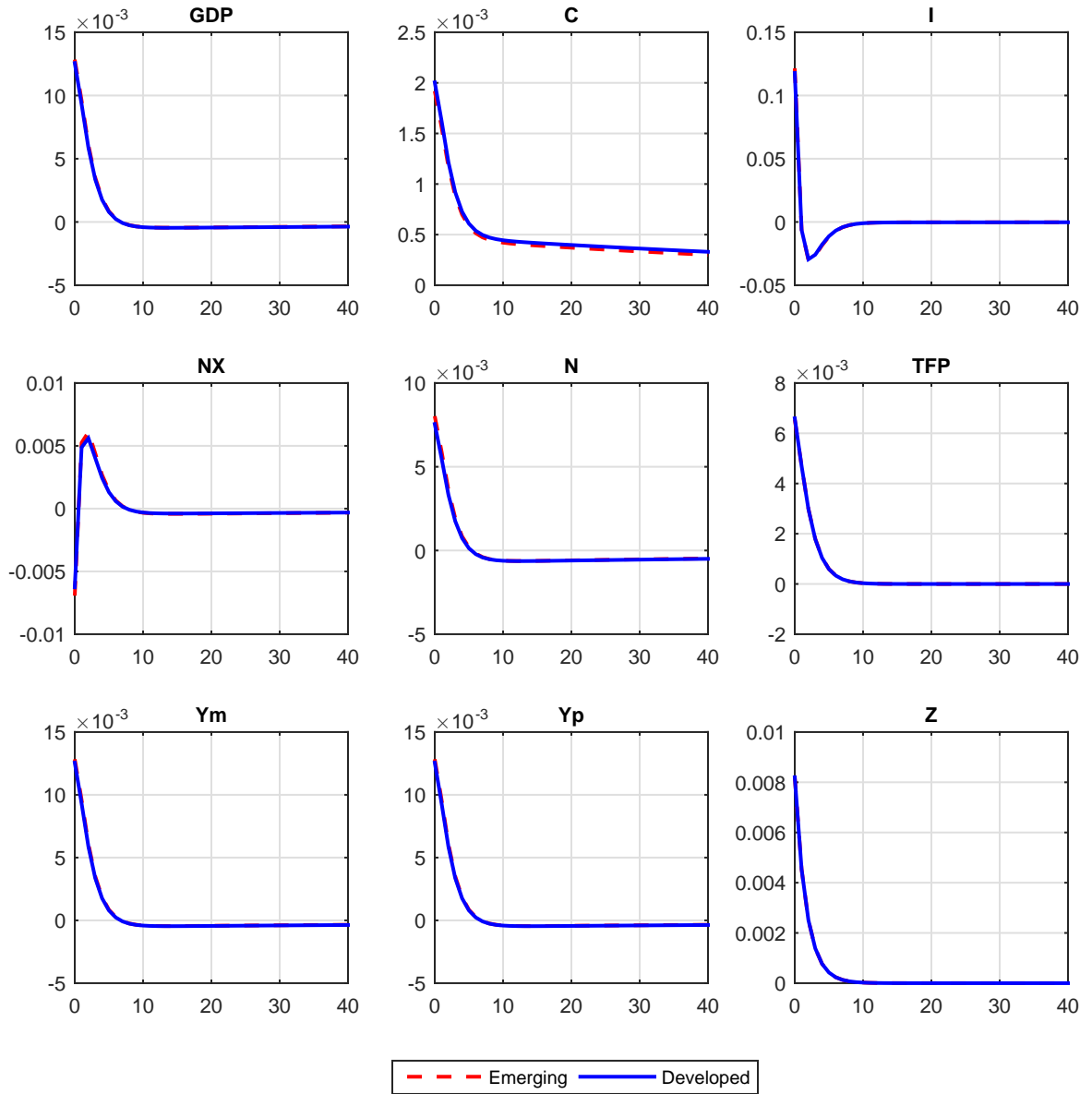
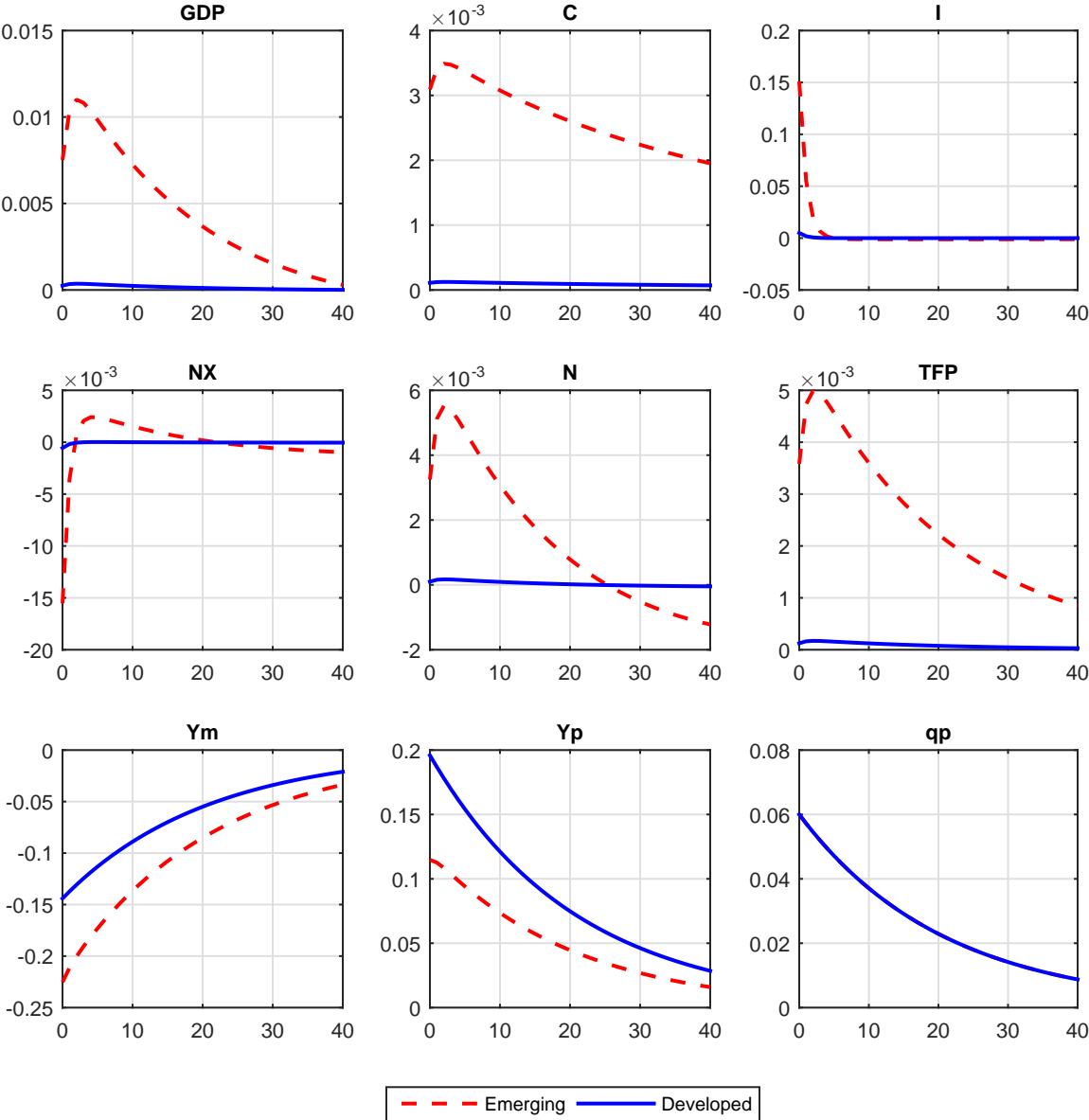


Figure 2: Impulse Response to a One-Standard-Deviation Shock to Price of Commodities



trade balance of a developed economy, but the net exports of manufactures to GDP of a typical emerging economy.

Panel A reports the moments that we target and the calibrated parameters for each of the economies. As documented in section 2.3, the average emerging economy has a net deficit of manufactures equal to 13 percent of GDP, while the average developed economy has almost balanced trade in each sector.

Panel B of table 6 presents the volatility of GDP, the ratio of net exports to GDP, consumption, investment, labor, and total factor productivity. Notice that, in the “Emerging balanced” calibration, the volatility of GDP, net exports to GDP, and consumption decrease with respect to the emerging economy benchmark, and their volatility is almost as low as in the developed economy. Indeed, this calibration explains as much as 91 percent of the difference in the volatility of GDP between the developed and emerging economy in the model, 88 percent of net exports to GDP, and 75 percent of the difference in the volatility of consumption. That is, the higher volatility of aggregate variables in the emerging economy can be explained mostly by its larger sectoral trade imbalances. The comparison between a developed economy and a similar economy with the sectoral trade imbalances of an emerging economy confirms this intuition.

Panel C presents the statistics for correlation of each variable with GDP for each of the cases described above. In these cases, however, there are no large differences in the moments implied by the alternative calibrations.

Finally, panel D of table 6 shows the results for the autocorrelation of the variables for each of the calibrations. As is the case for volatility, the calibration for the “Emerging balanced” economy captures most of the difference in persistence between the developed and the emerging economy. This exercise shows that most of the difference between emerging and developed economies, in the model, stems from the different sectoral trade imbalances.

Table 6: Role of Country-Specific Parameters for Emerging Market Business Cycles

<i>A. Target moments and calibrated parameters</i>						
	Y_p/GDP	NX/GDP	NX_m/GDP	A_p	b	η
Emerging	0.67	-0.09	-0.13	1.11	1.33	0.45
Emerging balanced	0.67	-0.09	-0.01	1.11	1.41	0.37
Developed	0.42	-0.01	-0.01	0.96	0.13	0.55
Developed imbalanced	0.42	-0.01	-0.13	0.96	0.14	0.64

<i>B. Volatility</i>						
	Std. dev. (%)		Std. dev. relative to GDP			
	GDP	NX/GDP	C	I	N	TFP
Emerging	3.63	3.21	0.38	3.46	0.58	0.46
Emerging balanced	1.59	1.55	0.26	4.04	0.63	0.49
Developed	1.39	1.32	0.22	4.28	0.63	0.50
Developed imbalanced	4.55	3.73	0.42	3.39	0.55	0.48

<i>C. Correlation with GDP</i>						
	GDP	NX/GDP	C	I	N	TFP
Emerging	1.00	0.71	0.68	0.37	0.94	0.97
Emerging balanced	1.00	0.68	0.66	0.38	0.97	0.99
Developed	1.00	0.67	0.60	0.38	0.98	0.99
Developed imbalanced	1.00	0.70	0.64	0.37	0.92	0.97

<i>D. Autocorrelation</i>						
	GDP	NX/GDP	C	I	N	TFP
Emerging	0.72	0.41	0.86	-0.02	0.72	0.70
Emerging balanced	0.36	0.18	0.72	-0.31	0.36	0.36
Developed	0.25	0.07	0.57	-0.36	0.29	0.24
Developed imbalanced	0.76	0.40	0.87	0.03	0.77	0.75

Note: “Emerging balanced” is an economy with production structure and overall trade balance of an emerging country, but sectoral trade balance of a developed country. “Developed imbalanced” is an economy with production structure and overall trade balance of a developed country, but sectoral trade balance of an emerging country.

5 Conclusion

In this paper, we have documented two key differences between emerging and developed countries that have not received much attention in the literature on emerging market business cycles. The first difference is that the production of commodities accounts for a much larger share of value added in emerging than in developed countries. The second difference is that aggregate trade deficits in emerging economies are larger, driven by large deficits in manufactured goods.

We have calibrated a small open economy with two sectors to capture these differences, and we have shown that they result in business cycles that are considerably more volatile in emerging countries. In the data, the volatility of both GDP and net exports as a share of GDP are much larger in emerging economies, and this pattern is captured by the model. These results are driven by sectoral trade imbalances that arise from the mismatch between the structure of production and of consumption in emerging countries, which lead to high exposure to the international relative price of commodities. Shocks to this relative price therefore serve as a significant driver of emerging market business cycles. Thus, the volatility of international commodity prices leads to volatile business cycles in emerging economies, even in a model where these countries face exactly the same processes for productivity and relative prices as developed countries.

Our results provide new insights into the drivers of the high volatility of emerging market business cycles. We find that terms of trade shocks have a significant impact on the business cycle dynamics of emerging economies because they consume more manufactured goods than they produce. These findings have implications for the design of trade and fiscal policy in these countries.

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Appendix: Equilibrium conditions

The equilibrium conditions include the first-order conditions of the households, the manufacturing and primary goods producers, and the producer of final goods. In addition, there are the laws of motion for bond holdings, capital, productivity shocks and the relative price of primary goods, the production functions, and the market clearing conditions.

Households

The households' first-order conditions for consumption of the final good and labor supply to the two sectors are

$$\alpha C_t(s^t)^{\alpha(1-\gamma)-1} (1 - N_t(s^t))^{(1-\alpha)(1-\gamma)} = \lambda_t(s^t) p_t(s^t) \quad (21)$$

$$\frac{1 - \alpha}{\alpha} \frac{C_t(s^t)}{1 - N_t(s^t)} = \frac{w_t(s^t)}{p_t(s^t)} \quad (22)$$

where $\lambda_t(s^t)$ is the Lagrange multiplier on the household's budget constraint (2).

The first-order condition with respect to the future capital stock delivers the standard equation for pricing returns

$$1 = \mathbb{E}_t [m_{t+1}(s_{t+1}) r_{t+1}^K(s_{t+1}) | s^t] \quad (23)$$

where the stochastic discount factor $m_{t+1}(s_{t+1} | s^t)$ is

$$m_{t+1}(s_{t+1} | s^t) = \beta \left(\frac{C_{t+1}(s_{t+1} | s^t)}{C_t(s^t)} \right)^{\alpha(1-\gamma)-1} \left(\frac{1 - N_t(s_{t+1} | s^t)}{1 - N_t(s^t)} \right)^{(1-\alpha)(1-\gamma)} \quad (24)$$

and $r_{t+1}^K(s_{t+1} | s^t)$ is the return on capital investment given by

$$r_{t+1}^K(s_{t+1} | s^t) = \left(\phi \left(\frac{K_{t+1}(s^t)}{K_t(s^{t-1})} - 1 \right) + 1 \right)^{-1} \times \left(\frac{r_{t+1}(s_{t+1} | s^t)}{p_{t+1}(s_{t+1} | s^t)} + (1 - \delta) + \frac{\phi}{2} \left[\left(\frac{K_{t+2}(s_{t+1} | s^t)}{K_{t+1}(s^t)} \right)^2 - 1 \right] \right) \quad (25)$$

The remaining first-order condition with respect to the choice of non-contingent bonds, delivers the following expression for the bond price:

$$q_t(s^t) = \mathbb{E}_t [m_{t+1}(s_{t+1}) | s^t] \quad (26)$$

Firms

The final goods firm's first-order conditions with respect to the demand for manufacturing and primary goods are for $i \in \{m, p\}$:

$$\frac{\partial G(X_{m,t}(s^t), X_{p,t}(s^t))}{\partial X_{i,t}(s^t)} = \frac{q_{i,t}(s^t)}{p_t(s^t)} \quad (27)$$

In the manufacturing and primary goods sectors, firms' first-order conditions for labor and capital imply:

$$w_t(s^t) = \theta_n Z_t(s^t) \frac{K_{m,t}(s^{t-1})^{\theta_k} N_{m,t}(s^t)^{\theta_n}}{N_{m,t}(s^t)} \quad (28)$$

$$= \theta_n q_{p,t}(s^t) A_p Z_t(s^t) \frac{K_{p,t}(s^{t-1})^{\theta_k} N_{p,t}(s^t)^{\theta_n}}{N_{p,t}(s^t)} \quad (29)$$

$$r_t(s^t) = \theta_k Z_t(s^t) \frac{K_{m,t}(s^{t-1})^{\theta_k} N_{m,t}(s^t)^{\theta_n}}{K_{m,t}(s^{t-1})} \quad (30)$$

$$= \theta_k q_{p,t}(s^t) A_p Z_t(s^t) \frac{K_{p,t}(s^{t-1})^{\theta_k} N_{p,t}(s^t)^{\theta_n}}{K_{p,t}(s^{t-1})} \quad (31)$$

These conditions imply that

$$\frac{K_{m,t}(s^{t-1})}{N_{m,t}(s^t)} = \frac{K_{p,t}(s^{t-1})}{N_{p,t}(s^t)} \quad (32)$$

The shares of capital and labor in production in the two sectors are the same and both sectors face the same wage and rental rate of capital, so firms in both sectors choose the same capital labor ratio.

The conditions also imply that

$$\frac{Y_{m,t}(s^t)}{q_{p,t}(s^t) Y_{p,t}(s^t)} = \frac{K_{m,t}(s^{t-1})}{K_{p,t}(s^{t-1})} = \frac{N_{m,t}(s^t)}{N_{p,t}(s^t)} \quad (33)$$

Remaining conditions

The remaining conditions necessary to solve for equilibrium are the laws of motion of capital (4), productivity (12), and the relative price of primary goods (13); and the market clearing conditions (14), (15), (16), and (17).